

Uplink CDMA

user 1	wants to transmit	$14 = s_1$.. send	$14 \times c_1 = 14 \times [1 \ 1 \ 1 \ 1]$
" 2	" "	$20 = s_2$		$20 \times [1 \ 1 \ -1 \ -1]$
" 3	" "	$26 = s_3$		$26 \times [1 \ -1 \ -1 \ 1]$
" 4	" "	$-5 = s_4$		$-5 \times [1 \ -1 \ 1 \ -1]$

@ BS, $\underline{r} = [55 \ 13 \ -37 \ 25]$

↑ user i will transmit $s_i c_i$

$$\underline{r} = \sum_i s_i c_i + \underbrace{p}_0$$

At the BS (receiver),

to recover s_3 ,

let's try to compute

$$\begin{aligned} \langle \underline{r}, c_3 \rangle &= \underline{r} \cdot c_3 = \left(\sum_i s_i c_i \right) \cdot c_3 \\ &= \sum_i s_i c_i \cdot c_3 = s_3 \underbrace{c_3 \cdot c_3}_{[1 \ -1 \ -1 \ 1] \cdot [1 \ -1 \ -1 \ 1]} \\ &= 4 s_3 \end{aligned}$$

$$\hat{s}_3 = \frac{1}{4} \langle \underline{r}, c_3 \rangle$$

↓

$$\hat{s}_3 = \frac{1}{4} \langle \underline{r}, \underline{c}_3 \rangle$$



$$\hat{s}_i = \frac{1}{4} \langle \underline{r}, \underline{c}_i \rangle$$

MATLAB

Define the code matrix

$$C = \begin{bmatrix} \underline{c}_1 \\ \underline{c}_2 \\ \underline{c}_3 \\ \underline{c}_4 \end{bmatrix}$$

Note that $CC^T = 4 \times I = N \times I$

$$\begin{bmatrix} \underline{c}_1 \\ \underline{c}_2 \\ \underline{c}_3 \\ \underline{c}_4 \end{bmatrix} \times \begin{bmatrix} \underline{c}_1^T \\ \underline{c}_2^T \\ \underline{c}_3^T \\ \underline{c}_4^T \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = 4I$$

Define message (row) vector

$$\underline{s} = [s_1 \ s_2 \ s_3 \ s_4]$$

The received (row) vector

$$\underline{r} = \sum_i s_i \underline{c}_i = [s_1 \ s_2 \ s_3 \ s_4] \begin{bmatrix} \underline{c}_1 \\ \underline{c}_2 \\ \underline{c}_3 \\ \underline{c}_4 \end{bmatrix}$$

To recover s_j , we compute

$$\hat{s}_j = \frac{1}{4} \langle \underline{r}, \underline{c}_j \rangle = \frac{1}{4} \underline{r} \underline{c}_j^T$$

$$\begin{aligned} \hat{\underline{s}} &= [\hat{s}_1, \hat{s}_2, \hat{s}_3, \hat{s}_4] = \left[\frac{1}{4} \underline{r} \underline{c}_1^T, \frac{1}{4} \underline{r} \underline{c}_2^T, \frac{1}{4} \underline{r} \underline{c}_3^T, \frac{1}{4} \underline{r} \underline{c}_4^T \right] \\ &= \frac{1}{4} \underline{r} \begin{bmatrix} \underline{c}_1^T & \underline{c}_2^T & \underline{c}_3^T & \underline{c}_4^T \end{bmatrix} \\ &= \frac{1}{4} \underline{r} \underline{C}^T \end{aligned}$$

Conclusion : start with $\underline{s}, \underline{C}$

$$T_x : \underline{s} \underline{C} \rightarrow \underline{r}$$

$$R_x : \frac{1}{4} \underline{r} \underline{C}^T \rightarrow \hat{\underline{s}}$$

check: $\frac{1}{4} \underline{r} \underline{C}^T = \frac{1}{4} (\underline{s} \underline{C}) \underline{C}^T = \frac{1}{4} \underline{s} 4 \mathbf{I} = \underline{s}$

Downlink

BS \rightarrow MS's

BS knows s_1, s_2, \dots, s_N

$\underline{c}_1, \underline{c}_2, \dots, \underline{c}_N$

$$\text{Transmit } \underline{s} \underline{C} = \sum_i s_i \underline{c}_i$$

Each MS

Receives $\underline{r} = \underline{s} \underline{C} + \text{noise}$

i^{th} MS recovers s_i by

$$\frac{1}{N} \underline{r} \underline{c}_i^T$$

only need its own code.

Uplink

MS's \rightarrow BS

i^{th} MS only knows s_i, \underline{c}_i

Transmit $s_i \underline{c}_i$

these signals from different users are combined in the air.

$$\underline{r} = \sum_i s_i \underline{c}_i = \underline{s} \underline{C}$$

BS recovers

$$\hat{\underline{s}} = \frac{1}{N} \underline{r} \underline{C}^T$$

need to know all \underline{c}_i